NEAR-OPTIMAL SOURCE PLACEMENT FOR LINEAR PHYSICAL FIELDS

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ABSTRACT

In real-word applications, signal processing is often used to measure and control a physical field by means of sensors and sources, respectively. An aspect that has been often neglected is the optimization of the sources' locations. In this work, we discuss the source placement problem as the dual of the sensor placement problem and propose two polynomial-time algorithms, for scenarios with or without noise. Both algorithms are near-optimal and indicate the possibility to make the control of such physical fields easier, more efficient and more stable to noise.

Index Terms— Sensor placement, Inverse problem, Source placement

1. INTRODUCTION

Many real-world signal processing problems involve the sensing and the control of a physical field. A simple example is the temperature in a building: we measure it with a sensor network and we control the sources, i.e. the heaters, to have a desired temperature distribution. In these scenarios, we face several joint problems, such as:

- the control of the physical field,
- the sampling of the physical field in certain locations using a set of sensors,
- the reconstruction of the physical field from the measurements.

These problems already received significant attention in the literature because of their fundamental role. However, there are two aspects that are often neglected and may significantly impact the performance of the system: the optimization of the sensors' locations, to improve the reconstruction of the physical field from the measurements, and of the sources' locations, to improve the control of the physical field itself. While the first problem has recently received some attention [1–4], the second one is rarely discussed in the literature.

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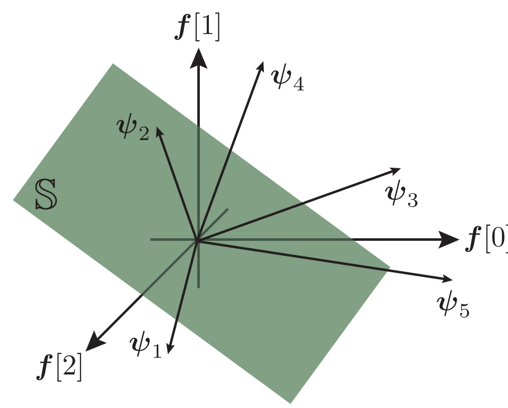

Fig. 1. A graphical representation of the source placement problem where $\mathbb{S}$ is a 2-dimensional subspace of $\mathbb{R}^3$ and we have to choose between 5 source indicated by the vectors $\boldsymbol{\psi}$.

1.1. Problem Statement

In this paper, we state the source placement problem as the dual of the sensor placement problem, that is described in [2]. More precisely, we consider a discrete physical field $\boldsymbol{f} \in \mathbb{R}^N$ and we define a K-dimensional subspace $\mathbb{S} \subseteq \mathbb{R}^N$ representing all the $\boldsymbol{f}$ that we would like to enforce with the sources. We consider a set of M sources' positions and each source acts linearly on the physical field. Therefore, we model the source-field relationship as,

$$\boldsymbol{f} = \boldsymbol{\Psi}\boldsymbol{\alpha}, \tag{1}$$

where $\boldsymbol{\alpha} \in \mathbb{R}^M$ are the sources' intensities and $\boldsymbol{\Psi} \in \mathbb{R}^{N\times M}$ is a linear model representing the action of the sources. A graphical representation of the problem is given in Figure 1.

Now, assume that we cannot use all the sources and we must choose a subset $\mathcal{L}$ of L sources to represent the subspace $\mathbb{S}$. We obtain a pruned matrix $\boldsymbol{\Psi}_{\mathcal{L}} \in \mathbb{R}^{N\times L}$ and a pruned source vector $\boldsymbol{\alpha}_{\mathcal{L}} \in \mathbb{R}^L$,

$$\boldsymbol{f} = \boldsymbol{\Psi}_{\mathcal{L}}\boldsymbol{\alpha}_{\mathcal{L}}, \tag{2}$$

where the subscript $\mathcal{L}$ indicates that we kept the sources indexed by $\mathcal{L}$. Two questions regarding the choice of $\mathcal{L}$ arise:

1. How do we choose the L sources?

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2. Which cost function shall we consider to evaluate the quality of the chosen subset?

First, we note that the problem of source selection is equivalent to choose a set of L columns from $\mathbf{\Psi}$ such that the desired cost function, usually related to the spectrum of $\mathbf{\Psi}$, is optimized. Then, we note that source selection is intrinsically combinatorial, as many other subset selection problems. In fact, we need to test all the $\binom{N}{L}$ possible sources subsets to find the optimal one. Therefore, we look for an approximation algorithm that reaches a sub-optimal solution with guaranteed quality and computable in polynomial time. The guarantee is often expressed with the concept of *near-optimality* and is measured by the *approximation factor*, a multiplicative factor bounding the worst-case distance from the optimal solution. For example, a minimization algorithm with an approximation factor of 2 always generates a solution whose cost function is at most two times larger than the optimal solution.

In what follow, we identify two different scenarios differentiated by the presence of noise. As in other signal processing problems, such difference leads to different algorithms, considered cost functions and results.

1.1.1. Noiseless source placement

If the target field $\boldsymbol{f}$ is noiseless, we recall the source placement problem as finding the set of columns of $\mathbf{\Psi}$ that spans the subspace that approximates $\mathbb{S}$ as precisely as possible. In other words, we have the following problem.

Problem 1. *Consider a K-dimensional subspace $\mathbb{S} \subseteq \mathbb{R}^N$ and a matrix $\mathbf{\Psi} \in \mathbb{R}^{N\times M}$. Given the number of sources L, find the source placement $\mathcal{L}$ such that the error $\mathbf{E}_{\boldsymbol{f}}\left[\|\boldsymbol{f} - P_{\mathbf{\Psi}_\mathcal{L}}\boldsymbol{f}\|_2^2\right]$ is minimized, where $P_{\mathbf{\Psi}_\mathcal{L}}$ is the linear operator projecting $\boldsymbol{f}$ onto the subspace spanned by $\mathbf{\Psi}_\mathcal{L}$.*

Note that we must define a probabilistic distribution for $\boldsymbol{f}$. In this paper, we consider $\boldsymbol{f} = \mathbf{\Phi}_\mathbb{S}\boldsymbol{x}$, where $\mathbf{\Phi}_\mathbb{S}$ is a basis for the subspace $\mathbb{S}$ and $\boldsymbol{x}$ is a set of i.i.d. Gaussian random variables with unit variance.

1.1.2. Noisy source placement

Consider a given target field $\boldsymbol{f} = \mathbf{\Phi}_\mathbb{S}\boldsymbol{x}$ and a set of sources $\mathcal{L}$, where $\mathbf{\Phi}_\mathbb{S}$ is a basis for the K-dimensional subspace $\mathbb{S}$. Moreover, assume that an i.i.d. Gaussian noise $\boldsymbol{w}$ is corrupting $\boldsymbol{f}$ with a variance σ^2, due to reconstruction error of the actual state of the physical field. The noise perturbing $\boldsymbol{f}$ propagates to the estimated sources and complicates the control of the physical field.

We aim at finding the sources $\boldsymbol{\alpha}_\mathcal{L}$ such that the produced physical field $\widetilde{\boldsymbol{f}}$ minimizes the ℓ_2 distance $\|\boldsymbol{f} - \widetilde{\boldsymbol{f}}\|_2$.

If $L \geq K$ and $\operatorname{rank}(\mathbf{\Phi}_\mathbb{S}^\dagger\mathbf{\Psi}_\mathcal{L}) = K$, we estimate $\boldsymbol{\alpha}_\mathcal{L}$ as

$$\widetilde{\boldsymbol{\alpha}}_\mathcal{L} = \mathbf{\Psi}_\mathcal{L}{}^\dagger\boldsymbol{f} = (\mathbf{\Psi}_\mathcal{L}^*\mathbf{\Psi}_\mathcal{L})^{-1}\mathbf{\Psi}_\mathcal{L}^*\boldsymbol{f}, \tag{3}$$

where $\dagger$ indicates the Moore-Penrose pseudoinverse. Then, if $\mathbf{\Psi}_\mathcal{L}$ spans $\mathbb{S}$, the error in the generated field is equal to,

$$\|\boldsymbol{f} - \widetilde{\boldsymbol{f}}\|_2^2 = \sigma^2\sum_{i=1}^{K}\frac{1}{\lambda_i(P_\mathbb{S}\mathbf{\Psi}_\mathcal{L})} \tag{4}$$

where $\lambda_i(P_\mathbb{S}\mathbf{\Psi}_\mathcal{L})$ is the i-th eigenvalue of $P_\mathbb{S}\mathbf{\Psi}_\mathcal{L}P_\mathbb{S}{}^*\mathbf{\Psi}_\mathcal{L}{}^*$ and $P_\mathbb{S}$ is the projection operator onto the subspace $\mathbb{S}$. Finally, we characterize the noisy source placement problem as follows.

Problem 2. *Consider a physical system modeled as $\mathbf{\Psi} \in \mathbb{R}^{N\times M}$ and a subspace $\mathbb{S} \subseteq \mathbb{R}^N$. Assume that the target fields $\boldsymbol{f} \in \mathbb{S}$ are corrupted by i.i.d. Gaussian noise $\boldsymbol{w}$ with variance σ^2. Find the source location $\mathcal{L}$ such that* (4) *is minimized $\forall \boldsymbol{f} \in \mathbb{S}$.*

1.2. Contributions

In this paper, we present two algorithms that solve respectively Problem 1 and 2, have a polynomial complexity and are near optimal w.r.t. the chosen cost function. More precisely,

- for Problem 1, we propose an algorithm that greedily minizes the distance between the subspace achieved by the sources and $\mathbb{S}$ and is near-optimal in terms of the approximation error with the subspace spanned by $\mathbf{\Psi}_\mathcal{L}$,
- for Problem 2, we propose a greedy algorithm based on the frame potential [5] that, under given conditions on $\mathbf{\Psi}$ and $\mathbb{S}$, is near optimal w.r.t. (4).

1.3. Literature Review

Up to the author's knowledge, the problem of source placement is new and has never been studied, at least in the signal processing community. However, there are four problems that are closely related: sensor placement, dictionary learning, dictionary selection and dimensionality reduction.

In the first problem, we attempt to choose L rows from a matrix $\mathbf{\Psi} \in \mathbb{R}^{N\times M}$ with $N > L \geq M$ such that the recovery of a vector of parameters $\boldsymbol{\alpha} \in \mathbb{R}^M$ from $\boldsymbol{f} = \mathbf{\Psi}_\mathcal{L}\boldsymbol{\alpha}$ is more stable to noise. This problem is equivalent to finding the L rows that forms the $\mathbf{\Psi}_\mathcal{L}$ closest to a tight frame. The details of such a problem and near-optimal algorithmic solutions are given in [2]. Since it is very hard to optimize (4) due to the presence of many local minima with arbitrarily bad performance, the literature regarding the sensor selection problem is centered around the choice and the analysis of proxy functions that are both efficiently optimized by approximation algorithm and able to indirectly optimize (4). Example of proxy cost functions are mutual information [3], entropy [6], R^2 [7] and frame potential [2].
The sensor placement problem is extremely similar to the noisy source placement, the main difference being the possibility of the latter to define a subspace $\mathbb{S}$ instead of $\mathbb{R}^N$.

In dictionary learning, we learn a dictionary $\mathbf{\Psi}_\mathcal{L} \in \mathbb{R}^{N\times L}$ such that a set of P given test vectors $\mathcal{F} = \{\boldsymbol{f}_0, \boldsymbol{f}_1, \ldots, \boldsymbol{f}_P\}$

are precisely represented, usually under a sparse prior. Note that dictionary learning differentiates from source placement for two principal aspects: the reference is not a given subspace but a set of vectors $\mathcal{F}$ and we can freely optimize the dictionary elements without being forced to pick them from $\boldsymbol{\Psi}$. See [8] for a review of such topic.

In dictionary selection, we form a dictionary $\boldsymbol{\Psi}_{\mathcal{L}} \in \mathbb{R}^{N\times L}$ by choosing L columns out of the M available ones in $\boldsymbol{\Psi} \in \mathbb{R}^{N\times M}$ such that a set of P given test vectors $\mathcal{F} = \{\boldsymbol{f}_0, \boldsymbol{f}_1, \ldots, \boldsymbol{f}_P\}$ is precisely represented, usually under a sparse prior. Dictionary selection is very similar to source placement, the main difference being the optimization target: a subspace $\mathbb{S} \subseteq \mathbb{R}^N$ for source placement and a set of vectors $\mathcal{F}$ for the dictionary learning. The two options are equivalent when the elements of $\mathcal{F}$ are lying on $\mathbb{S}$. If this is not the case, we may require a different optimization strategy, such as one promoting a sparse representation. See [9] for a detailed problem statement and a state-of-the-art solution.

Another similar problems is dimensionality reduction, where we usually attempt to approximate a set of vectors $\mathcal{F}$ with a K-dimensional manifold. A classical example of such a technique is principal component analysis, where we find the K-dimensional linear subspace that minimizes the ℓ_2 error w.r.t. $\mathcal{F}$. Note that the noiseless source placement is a principal component analysis constrained to select the components from a matrix $\boldsymbol{\Psi}$ instead of learning them from the available data.

In this paper, we make extensive use of the theory studying the greedy optimization of submodular functions. In particular, we use the result of Nemhauser et al. [10] that proves the near-optimality of algorithms based on a greedy maximization procedure.

2. A NEAR-OPTIMAL ALGORITHM FOR THE NOISELESS SOURCE PLACEMENT

In this section, we present an approximation algorithm that solves Problem 1 in polynomial time while being near-optimal w.r.t. the average approximation error,

$$\mathbf{E}_{\boldsymbol{f}}\left[\|\boldsymbol{f} - P_{\boldsymbol{\Psi}_{\mathcal{L}}}\boldsymbol{f}\|_2^2\right], \tag{5}$$

where $P_{\boldsymbol{\Psi}_{\mathcal{L}}}$ is the projection operator on the subspace spanned by $\boldsymbol{\Psi}_{\mathcal{L}}$. Here, we assume $\boldsymbol{f} = \boldsymbol{\Phi}_{\mathbb{S}}\boldsymbol{x}$ with $\boldsymbol{x}$ being a set of i.i.d. random Gaussian variables with zero mean and unitary variance.

The proposed algorithm removes at every iteration the source that maximally reduces the approximation error (5) and its details are given in Algorithm 1. It is a near-optimal algorithm w.r.t. (5) and to prove such characteristic, we first show that the chosen cost function is supermodular, i.e. the negated cost function is submodular.

Proposition 1. *The cost function $F(\mathcal{A}) = \mathbf{E}_{\boldsymbol{f}}\left[\|\boldsymbol{f} - P_{\boldsymbol{\Psi}_{\mathcal{A}}}\boldsymbol{f}\|_2^2\right]$ is decreasing and supermodular with $\mathcal{A}$.*

Proof. First, we know that the rank of the projection operator $P_{\boldsymbol{\Psi}_{\mathcal{A}}}$ is equal to the rank of $\boldsymbol{\Psi}_{\mathcal{A}}$. Moreover, we can show that

$$F(\mathcal{A}) = \mathbf{E}_{\boldsymbol{f}}\left[\|P_{\perp,\boldsymbol{\Psi}_{\mathcal{A}}}\boldsymbol{f}\|_2^2\right] = \sum_{i=1}^{R} |\langle \boldsymbol{f}, \boldsymbol{v}_i\rangle|^2, \tag{6}$$

where $P_{\perp,\boldsymbol{\Psi}_{\mathcal{A}}}$ is the orthogonal projection on the complement of the space spanned by $\boldsymbol{\Psi}_{\mathcal{A}}$, $\boldsymbol{v}_i$ are the eigenvector of such projection and $R = N - \mathrm{rank}(\boldsymbol{\Psi}_{\mathcal{A}})$ is the dimension of the complement space to $\mathcal{A}$. Assume that an initial set $\mathcal{A}$ is given and consider i to be another element that we can add to $\mathcal{A}$. According to (6), we have two possible outcomes when we add a new element to $\mathcal{A}$:

- $\mathrm{rank}(\boldsymbol{\Psi}_{\mathcal{A}})$ increases by one, we loose the i^*-th eigenvector of $P_{\perp,\boldsymbol{\Psi}_{\mathcal{A}}}$ and $F(\mathcal{A})$ diminishes by $|\langle \boldsymbol{f}, \boldsymbol{v}_{i^*}\rangle|^2$.
- $\mathrm{rank}(\boldsymbol{\Psi}_{\mathcal{A}})$ and $F(\mathcal{A})$ do not change.

Note that these outcomes indicate that the function $F(\mathcal{A})$ is always decreasing and supermodular. In fact, the function is modular except when we add an element that does not increase $\mathrm{rank}(\boldsymbol{\Psi}_{\mathcal{A}})$, making it supermodular. □

We can use Proposition 1 jointly with the theorem of Nemhauser et al [10] to show the near-optimality of Algorithm 1.

Proposition 2. *Consider a matrix $\boldsymbol{\Psi} \in \mathbb{R}^{N\times M}$, a K-dimensional subspace $\mathbb{S}$ and assume that we want to find the L sources $\mathcal{L}$ minimizing $F(\mathcal{L})$. Then, Algorithm 1 is near-optimal and we can bound its performance as,*

$$\mathbf{E}_{\boldsymbol{x}}\left[\|\boldsymbol{f}\|_2^2\right] - F(\mathcal{L}) \le \left(1 - \frac{1}{e}\right)\left(\mathbf{E}_{\boldsymbol{x}}\left[\|\boldsymbol{f}\|_2^2\right] - \mathcal{F}(\mathit{OPT})\right),$$

where $\mathcal{L}$ and OPT are the set optimized by Algorithm 1 and by an optimal algorithm, respectively.

The proof follows from the result given in [10]. Note that Algorithm 1 has to be worst-out greedy algorithm to satisfy the conditions given in Nemhauser's theorem.

Finally, we explain how to compute $F(\mathcal{A})$ without testing all the infinite $\boldsymbol{f} \in \mathbb{S}$. Assume that $\boldsymbol{f} = \boldsymbol{\Phi}_{\mathbb{S}}\boldsymbol{x}$ and $\boldsymbol{x}$ are i.i.d. zero-mean Gaussian random variables with $\sigma^2 = 1$. Then, it is possible to prove the following equality,

$$F(\mathcal{A}) = \sum_{i\in\mathcal{A}}\sum_{j=1}^{R} |\langle \boldsymbol{\phi}_i, \boldsymbol{v}_j\rangle|^2, \tag{7}$$

where $\boldsymbol{\phi}_i$ is the i-th column of $\boldsymbol{\Phi}_{\mathbb{S}}$. In other words, it is just possible to test each column of $\boldsymbol{\Phi}_{S}$ to compute exactly $F(\mathcal{A})$.

3. A NEAR-OPTIMAL ALGORITHM FOR THE NOISY SOURCE PLACEMENT

If the target field $\boldsymbol{f}$ is corrupted by noise, we are trying to find a well-conditioned set of columns that spans $\mathbb{S}$ so that the MSE of $\boldsymbol{f}$ (4) is minimized.

Algorithm 1 Noiseless Source Placement

Require: Linear Model $\mathbf{\Psi}$, Number of sources L

Ensure: Sources locations $\mathcal{L}$

1. Initialize the available locations, $\mathcal{L} = \{1, \ldots, M\}$.
2. **Repeat until** L **locations are found**
   (a) If $|\mathcal{L}| = L$, stop.
   (b) Find the optimal column to remove, $i^* = \arg\max_{i \in \mathcal{L}} F(\mathcal{S} \cup i)$.
   (c) Update the available locations, $\mathcal{L} = \mathcal{L} \setminus i^*$.

It is clearly a harder problem, but we may use some interesting results derived in [2]. In particular, we derive an algorithm that selects the columns by a greedy worst-out minimization of the frame potential (FP), that is defined as

$$\mathrm{FP}(\mathbf{\Psi}_{\mathcal{A}}) = \sum_{i \in \mathcal{A}} \sum_{j \in \mathcal{A}} |\langle \boldsymbol{\psi}_i, \boldsymbol{\psi}_j \rangle|^2, \tag{8}$$

where $\boldsymbol{\psi}_i$ is the i-th column of $\mathbf{\Psi}$. The details of the algorithm are given in Algorithm 2 and in what follows we give some results showing its near-optimality in terms of the MSE. We do not prove the results here, given the limited amount of space and their similarity to the proofs given in [2].

First, we use the supermodularity of the FP, proved in [2], and the aforementioned result of Nemhauser et al. to show the near-optimality w.r.t. the FP.

Theorem 1. *Consider a matrix $\mathbf{\Psi} \in \mathbb{R}^{N \times M}$ and a given number of sources L, such that $M \leq L < N$. Denote the optimal set of locations as OPT $= \arg\max_{\mathcal{A} \subset \mathcal{N}, |\mathcal{A}| = L} \mathrm{FP}(\mathbf{\Psi}_{\mathcal{A}})$ and the greedy solution found by Algorithm 2 as $\mathcal{L}$. Then, $\mathcal{L}$ is near-optimal in a* FP *sense with approximation factor $\gamma = \left(1 + \frac{1}{e}\left(\mathrm{FP}(\mathbf{\Psi})\frac{K}{L_{MIN}{}^2} - 1\right)\right)$, where L_{MIN} is the sum of the norms of the L columns with the smallest norm.*

Second, we define the concept of (δ, L)-bounded frame, a sufficient condition to prove the near-optimality of the MSE, given the near-optimality of the FP.

Definition 1 ((δ, L)-bounded frame)**.** *Consider a matrix $\mathbf{\Psi} \in \mathbb{R}^{N \times M}$ where $N \leq L < M$. Then, we say that $\mathbf{\Psi}$ is (δ, L)-bounded if for every $\mathcal{A} \subseteq \mathcal{N}$ we have*

$$\bar{\lambda} - \delta \leq \lambda_i \leq \bar{\lambda} + \delta,$$

where $1 \leq i \leq K$, λ_i is the i-th eigenvalue of $\mathbf{\Psi}_{\mathcal{A}} \mathbf{\Psi}_{\mathcal{A}}^*$, $\bar{\lambda}$ is their average and $\bar{\lambda} > \delta \geq 0$.

Note that the concept of (δ, L)-bounded frames can be related to the notion of RIP matrices used in compressive sensing to guarantee the reconstruction of a sparse vector from a limited number of linear measurements [11]. Moreover, it is possible to prove that there exists random matrices satisfying such condition but it is much harder to prove the existence of deterministic matrices with such property.

We are now ready to state the near-optimality of Algorithm 2, when $\mathbf{\Psi}$ is a (δ, L)-bounded frame.

Theorem 2. *Consider a matrix $\mathbf{\Psi} \in \mathbb{R}^{N \times K}$ and $L \geq K$ sensors. Assume $\mathbf{\Psi}$ to be a (δ, L)-bounded frame, then the solution $\mathcal{L}$ of FrameSense is near-optimal w.r.t.* MSE *with an approximation factor η,*

$$\eta = \gamma \frac{(\bar{\lambda} + \delta)^2}{(\bar{\lambda} - \delta)^2} \frac{L_{MAX}}{L_{MIN}},$$

where η is the approximation factor of the MSE *and γ is the approximation factor of the* FP.

The proof follows the one given in [2], with the difference that we are selecting the columns instead of the rows.

Algorithm 2 Noisy Source Placement

Require: Linear Model $\mathbf{\Psi}$, Number of sources L

Ensure: Sources locations $\mathcal{L}$

1. Normalize the columns of $\mathbf{\Psi}$ to have unit-norm.
2. Initialize the available locations, $\mathcal{L} = \{1, \ldots, M\}$.
3. **Repeat until** L **locations are found**
   (a) If $|\mathcal{L}| = L$, stop.
   (b) Find the optimal column to remove, $i^* = \arg\max_{i \in \mathcal{L}} \mathrm{FP}\left(\mathbf{\Psi}_{\mathcal{L} \setminus i}\right)$.
   (c) Update the available locations, $\mathcal{L} = \mathcal{L} \setminus i^*$.

4. CONCLUSION

We studied the source placement problem as the dual of sensor placement: a column selection out of a linear model $\mathbf{\Psi}$. We defined two problems differentiated by the presence of noise and each problem optimizes a different cost function. For the noiseless case, we proposed an algorithm that approximate any given subspace $\mathbb{S}$ using the span of a set $\mathcal{L}$ of columns of $\mathbf{\Psi}$ and it is near-optimal w.r.t. the approximation error. For the noisy case, we proposed an algorithm, based on a previous work [2], that optimizes the frame potential as a proxy of the mean-square error. Such an algorithm is near optimal under a given condition on $\mathbf{\Psi}$, but it can only deal with the case $\mathbb{S} = \mathbb{R}^N$.

Future work will consider the extension of Algorithm 2 to a generic subspace. Such work will be centered on the definition of a cost function that promotes the choice of a subspace that is close to $\mathbb{S}$ and it is formed by a set of columns that are well-conditioned.

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